TU Clausthal

Master Thesis: Study of a TPFA scheme for the stochastic Allen-Cahn problem with constraint through numerical experiments

The damage in continuum media may be mathematically described by a stochastic Allen-Cahn equation (SAC) of the form $du + (\Psi - \Delta u)\,dt = g(u)\,dW(t)$ for some suitable function $g : \mathbb{R} \to \mathbb{R}$, and a Brownian motion $W = (W_t)_{t \geq 0}$. The solution u is the damage parameter, which is constrained to take values in the interval $[0, 1]$. The term Ψ in the equation may be understood as a Lagrange multiplier associated with this constraint. Thanks to the stochastic force on the right-hand side of the equation, microscopic effects such as crack opening and closure and micro-adhesion are taken into account in the macroscopic mathematical model. In [BBBLV17] it has been shown it there exists a unique pair (u, Ψ) solving the initial value problem for (SAC) on a bounded Lipschitz domain of $\mathbb{R}^d$ with homogeneous Neumann boundary conditions. In [BSVZ25], a space-time discretization by a finite-volume-scheme (FVS) has been developed for the problem studied in [BBBLV17]. It was shown that the sequence of approximate solutions from the FVS converges towards the unique solution (u, Ψ). For applications, it is not only necessary to have efficient numerical schemes to calculate solutions of (SAC), but also to understand the properties of these schemes. Due to the nonlinearity Ψ, the solutions of the FVS cannot be calculated exactly by the computer. The authors in [SZ26] propose a splitting method to calculate approximate solutions of the FVS and they have written a code in Python to calculate solutions of this splitting method explicitly. It turns out that the calculated approximate solutions do not only take values inside the interval $[0, 1]$.

Tasks

1.) Understand the stochastic Allen-Cahn problem introduced in [BBBLV17] and the numerical scheme introduced in [BSVZ25] and [SZ26].

2.) Analyze the code associated with the numerical scheme in [SZ26].

3.) Improve the code for faster and more efficient computation. Develop ideas how to improve the scheme in such a way that numerical solutions are forced to stay in $[0, 1]$.

Requirements

- Good knowledge in programming in Python.
- Good knowledge in analysis and probability theory. Basic knowledge in differential equations and/or partial differential equations is s appreciated, but not necessary.

General

This Master thesis can be written in English or German. If you are interested or if you have any questions, please contact Prof. Dr. Aleksandra Zimmermann and/or Dr. Niklas Sapountzoglou.

References

[BBBLV17] C. Bauzet, E. Bonetti, G. Bonfanti, F. Lebon and G.Vallet: A global existence and uniqueness result for a stochastic Allen-Cahn equation with constraint. Math. Methods Appl. Sci. 40 (14) (2017) 5241-5261

[BSVZ25] C. Bauzet, C. Sultan, G. Vallet and A. Zimmermann: Theoretical analysis of a finite-volume scheme for a stochastic Allen-Cahn problem with constraint. Nonlinear Analysis, Volume 259, 2025, 113812, https://doi.org/10.1016/j.na.2025.113812

[SZ26] Niklas Sapountzoglou, Aleksandra Zimmermann. Study of a TPFA scheme for the stochastic Allen-Cahn problem with constraint through numerical experiments arXiv:2512.17712 [math.NA]. https://doi.org/10.48550/arXiv.2512.17712