TU Clausthal

Master Thesis: A computational study of a fully discrete scheme for the stochastic heat equation

In the theory of partial differential equations, the heat equation $\partial_t u = \Delta u$ is one of the most common equations. Existence and uniqueness of solutions, as well as other properties are well known in the literature. Adding uncertainties or random effects to the model, the heat equation becomes the stochastic heat equation (SHE) $du - \Delta u\, dt = g(u)\, dW$ for some suitable function $g : \mathbb{R} \to \mathbb{R}$ and a Brownian motion $W = (W_t)_{t\geq 0}$. Existence and uniqueness of solutions to (SHE) are well known, see, e.g., [DPZ14]. For applications, it is not only necessary to have efficient numerical schemes to calculate solutions of (SHE), but also to understand the properties of the schemes, such as convergence rates. A fully discrete numerical scheme for (SHE) on a bounded polygonal domain of $\mathbb{R}^3$ or $\mathbb{R}^2$ with homogeneous Neuman boundary conditions, of finite volume (FV) type in space and semi-implicit in time, has been proposed in [BNSZ24]. Recently, the authors of [SZ251] studied convergence rates for the numerical scheme introduced in [BNSZ24]. This paper also provides numerical experiments using a self-written code in Python, which is published in [SZ252].

Tasks

1.) Understand and explain the numerical scheme in [BNSZ24] as well as the error bounds given in [SZ251].

2.) Understand and analyze the code in [SZ252].

3.) Since the numerical scheme has to be computed for a large number of paths individually, the calculation time is very high. Improve the code for faster and more efficient computation. Compare computing time of the code in [SZ252] and your improved code.

4.) Run numerical experiments and give an interpretation of your results.

Requirements

- Good knowledge of programming in Python.
- Good knowledge in analysis and probability theory. Basic knowledge in stochastic differential equations and/or partial differential equations is appreciated, but not necessary.

General

This master thesis can be written in English or German. If you have any questions, please contact Prof. Dr. Aleksandra Zimmermann and/or Dr. Niklas Sapountzoglou.

References

[BNSZ24] Caroline Bauzet, Flore Nabet, Kerstin Schmitz, Aleksandra Zimmermann. Convergence of a finite-volume scheme for a heat equation with a multiplicative Lipschitz noise. ESAIM: Math. Model. Numer Anal. **57** (2023), no. 2, 745-783.

[DPZ14] G. Da Prato and J. Zabczyk. *Stochastic equations in infinite dimensions*, volume 152 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, second edition, 2014.

[SZ251] Niklas Sapountzoglou, Aleksandra Zimmermann. Convergence rates for a finite volume scheme of the stochastic heat equation. Comput. Methods Appl. Math. **25** (2025), no. 4, 981-1002.

[SZ252] Niklas Sapountzoglou, Aleksandra Zimmermann. Convergence-rates-for-a-finite-volume-scheme-of-the-stochastic-heat-equation. `https://github.com/NiSa4242/Convergence-rates-for-a-Finite-Volume-Scheme-of-the-stochastic-heat-equation`. 2025