TU Clausthal

Master Thesis: A computational study of a fully discrete scheme for the stochastic heat equation

In the theory of partial differential equations, the heat equation $\partial_t u = \Delta u$ is one of the most common equations. Its solvability and properties are well known in the literature. Adding uncertainties or random effects to the model, the heat equation becomes the stochastic heat equation (SHE) $du - \Delta u\, dt = g(u)\, dW$ for some suitable function $g : \mathbb{R} \to \mathbb{R}$ and a Brownian Motion $W = (W_t)_{t \geq 0}$. Properties of solutions to the SHE are well known, in particular in the framework of variational solutions (see, e.g., [1]). For applications, it is not only necessary to have efficient numerical schemes to calculate solutions of SHE, but also to understand the properties of the schemes, such as convergence rates. The convergence of a so-called finite volume scheme (FVS) of the SHE with homogeneous Neuman boundary conditions has been proven in [2]. Recently, the authors of [3] provided an analytical proof for convergence rates of a FVS for the SHE with homogeneous Neumann boundary conditions. This paper also provides numerical results using a self-written python-code to estimate the computational error which is published in [4].

Tasks

1.) You should understand and explain the numerical scheme in [3] and the steps of the analytical proof of its convergence rates. Reproduce the main result and the proof in your own way.

2.) Understand the python-code in [4].

3.) Since the numerical scheme has to be computed for a large number of paths individually, the calculation time is very high. Improve the code for faster and more efficient computation. Compare computing time of the code in [4] and your improved code.

4.) Run numerical experiments and give an interpretation of your results.

Requirements

- Good knowledge in Analysis and Probability Theory.
- Good knowledge in programming in python.
- Basic knowledge in stochastic differential equations and/or partial differential equations is appreciated, but not necessary.

General

This Master Thesis can be written in English or German. If you have any questions, please contact Prof. Dr. Aleksandra Zimmermann and/or Dr. Niklas Sapountzoglou.

Literatur

[1] Wei Liu, Michael Röckner. *Stochastic Partial Differential Equations: An Introduction.* Universitext, Springer, 2015.

[2] Caroline Bauzet, Flore Nabet, Kerstin Schmitz, Aleksandra Zimmermann. Convergence of a finite-volume scheme for a heat equation with a multiplicative Lipschitz noise. ESAIM: Math. Model. Numer Anal. **57** (2023), no. 2, 745-783.

[3] Niklas Sapountzoglou, Aleksandra Zimmermann. Convergence rates for a finite volume scheme of the stochastic heat equation. Comput. Methods Appl. Math. **25** (2025), no. 4, 981-1002.

[4] Niklas Sapountzoglou, Aleksandra Zimmermann. Convergence-rates-for-a-finite-volume-scheme-of-the-stochastic-heat-equation. `https://github.com/NiSa4242/Convergence-rates-for-a-Finite-Volume-Scheme-of-the-stochastic-heat-equation`. 2025