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An Efficient Learning Method for RBF Neural Networks

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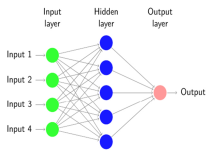
Abstract

Radial Basis Functions Neural Network (RBFNN) [1] as the outcome of recent research provides a simple model for complex networks. This is achieved by employing the Radial Basis Function (RBF) in the network as pattern. The optimal properties of the RBFs pave the way for stable approximation. However, it is generally rather difficult to determine the locations of the centers and the shape parameter. An evolutionary algorithm has been applied to learn the parameters. The approach is based on genetic algorithms.

Radial Basis Function Neural Networks

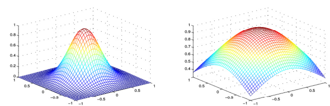
RBFNN has been applied successfully in many fields, including function approximation, chaotic time-series modeling [2], and data fusion. RBFNN can be introduced as a two-layer feedforward network, where the hidden neuron activation functions are RBFs.



RBF is a *multivariate* function $\Phi : \mathbb{R}^s \to \mathbb{R}$, such that

$$\Phi(x, x^c) = \phi(r \cdot \|x - x^c\|); \ \phi : [0, \infty) \to \mathbb{R}$$

where x^c is the *center point* and r is the radius. RBFs reach their maximum value when applied to their center points, and decline when applied to points far away from the center points. The shape parameter controls how distance affects the decrements; left $r = 1$ and right $r = 1/3$



The main goal is to minimize the point-wise estimation error, by **Power Function(PF)** [3] on centers $\mathcal{X} = \{x_1^c, \ldots, x_N^c\} \subseteq \Omega$ which result in

$$|o(x) - y(x)| \leq P_{\Phi,\mathcal{X}}(x) \|y\|_{\mathcal{N}_\Phi(\Omega)}, \forall x \in \Omega,$$

$$P_{\Phi,\mathcal{X}}(x) = \|\Phi(x, x) - \sum_{i=1}^{N} u_i(x)\Phi(x, x_i^c)\|_{\mathcal{N}_\Phi(\Omega)}$$

the error bound measures the smoothness of the data and the Power function value which is dependent on the distribution of the data set.

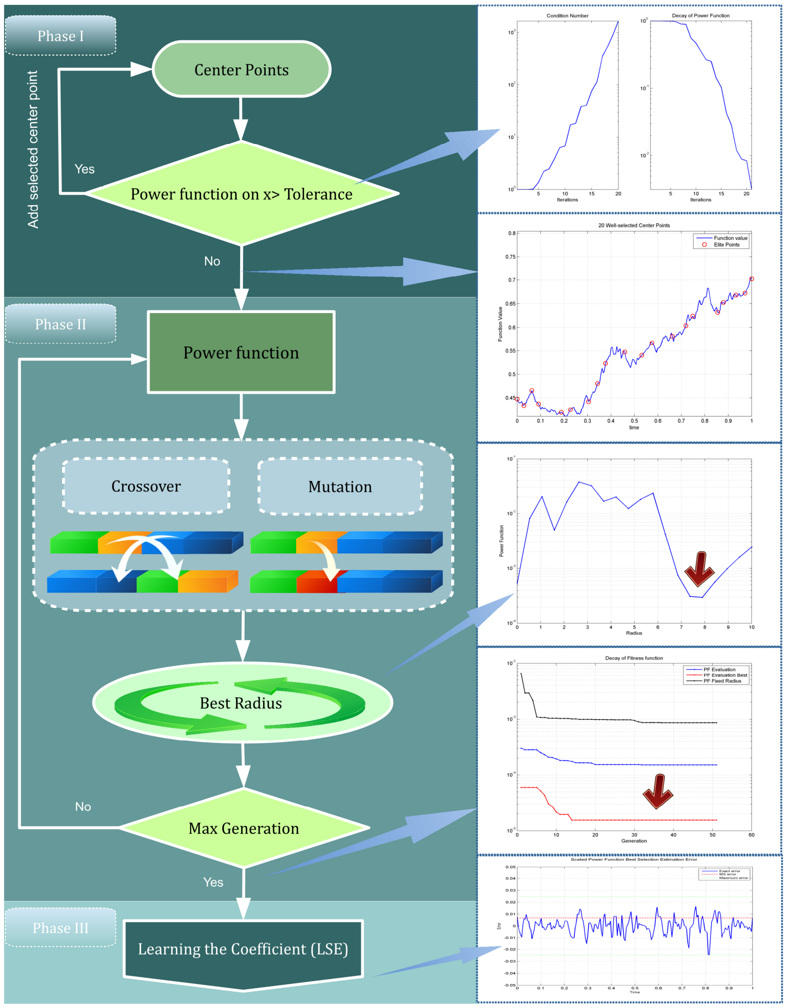
Main Objectives

The number and placement of the hidden neurons, centers, and also the value of the radius critically affects the results.

- **Selection of the center points (number and value)**:
  - **many centers** ⇒ model not suitable for prediction (overfitting), complexities and ill-conditioning.
  - **few centers** ⇒ poor approximation results.
- **Selection of the radius**:
  - **small radius** ⇒ overfitting.
  - **wider radius** ⇒ loose information and poor results.

Search Algorithm

- **PhaseI:** Learn the **number of center points** by an iterative deterministic **Greedy Search** based on the Power Function [4];
- **PhaseII:** Learn the **center points** x_i^c and the **radius** r by a **Genetic Algorithm**;
- **PhaseIII:** Calculate the **weights** in output $o(x) = \sum_{i=1}^{N} w_i \Phi(x, x_i^c)$ on well–selected center points and radius by Least Square Error (LSE).

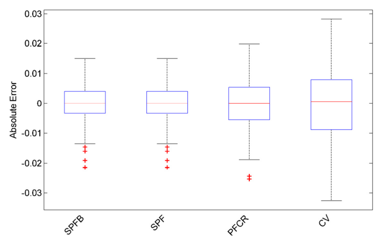


Results

The algorithm provides 20 centers, and compares PF and Cross Validation(CV) in order to select the centers. The best radius will be selected for every generation.

Case	Max error	MS error	Mean Radii	Optimal Radii
PF Scaled Radius	$2.4e-02$	$6.4e-03$	6.79	7.89
Cross Validation	$3.3e-02$	$1.1e-02$	2.25	–

The estimation evaluation based on PF provides better results compared to CV. Obviously, PF evaluation with fixed random radius is not consistent. The error evaluation based on PF(Best and with fixed radius) and CV by best parameters found in algorithm after 5 trials.



Forecasting based on RBFNNs highly depends on the efficient pruning of the parameters. Focusing on objective parameters scenario, genetic algorithm provides a better search strategy based on the new fitness function "Power function".

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[2] J. Frank, N. Davey and P. Hunt, *Time series Prediction and Neural Networks*, Hatfield, UK, 1997.
[3] Z. Wu, R. Schaback, *Local error estimates for radial basis function interpolation of scattered data*, IMA J.Numer. Anal. 13, 13-27, 1993.
[4] M. Pazouki, R. Schaback, *Bases for Kernel Based Spaces*, Computational and Applied Mathematics, 236:575-588, 2011.