TU Clausthal

Simulation and Optimization

Model Based Search for Combinatorial Optimization

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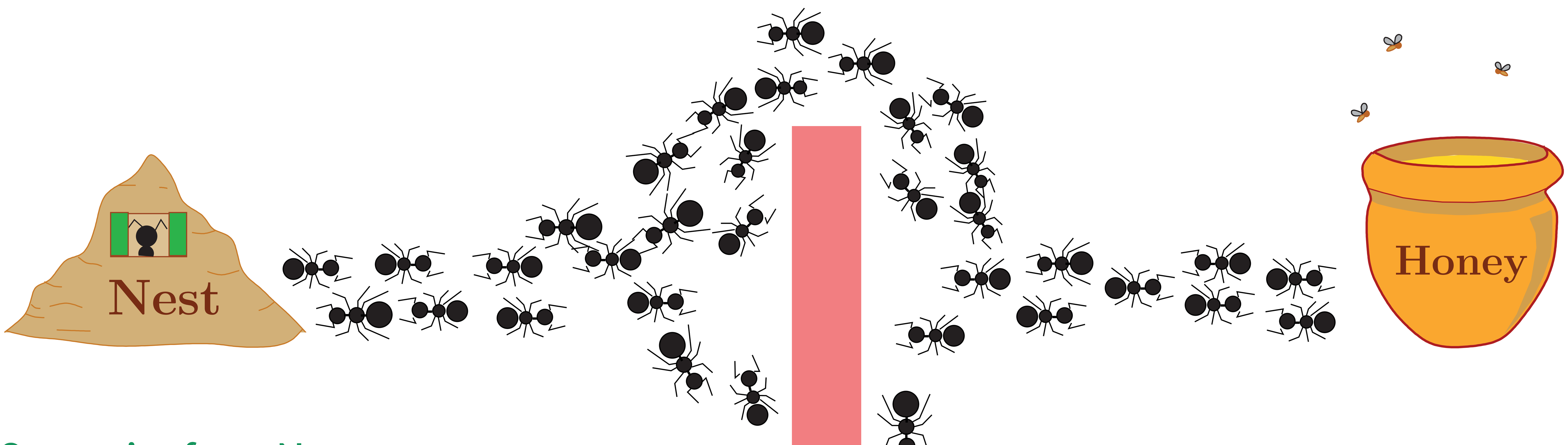

Strategies from Nature

Today, many heuristic solution techniques for complex combinatorial optimization problems adopt ideas from nature. Ant algorithms, e.g. imitate the foraging behaviour of ants that are able to find shortest paths in a difficult terrain by depositing and detecting pheromones.
For many problems e.g. network design for traffic and logistics, excellent solutions can be found by simulating the random search of artificial ants. Instead of pheromones, a central stochastic model Π_t is used that is adapted to successful steps in the search.

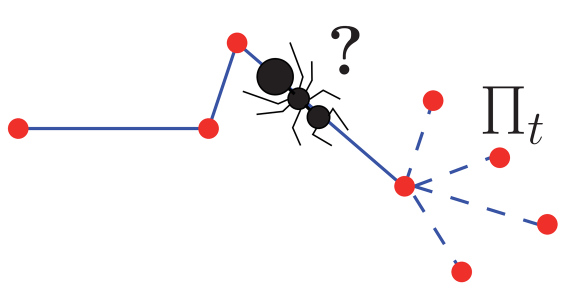

Figure 1: Each solution (path) is built sequentially, arcs are drawn according to a probability provided by the present model Π_t

Mathematical Model:

Model Based Search

As a generalization of ant algorithms, the model based search (MBS) paradigm was introduced. The *model* Π_t is a probability measure on the elements from which solutions are built. In the t-th iteration, samples of solutions are drawn according to Π_t, see Fig. 1 for an example, where a solution is a path. The quality of these solutions is evaluated and statistical inference is applied to the *good* solutions in the sample resulting in a temporary model W_t that reflects properties of good solutions, e.g. short paths.
The central model is now updated as $\Pi_{t+1} = (1-\rho_t)\Pi_t + \rho_t W_t$, where ρ_t is the *learning rate*. ρ_t determines how much the new knowledge W_t will influence future behaviour. Different from earlier models, [1] introduces an additional feasibility function that allows a more subtle control of how solutions are drawn from the model.
The MBS covers and generalizes many different heuristic optimization methods, besides ant algorithms these are e.g., cross entropy optimization, estimation of distribution algorithms and certain types of genetic algorithms.

Results:

How to Avoid 'Premature' Convergence

- In order to reach optimal solutions in finitely many iterations with probability one, we must use learning rates ρ_t that may become arbitrarily small, formally $\liminf_{t\to\infty} \rho_t = 0$.
- In [1] a new type of 'premature' convergence of the algorithm was detected: with a fixed learning rate $\rho > 0$, the sampling will produce just a single solution s_∞ after finitely many iterations, and s_∞ need not be optimal.
- Convergence of the model $\lim_{t\to\infty} \Pi_t = \Pi_\infty$ is desirable, if Π_∞ is concentrated on optimal solutions. This can be guaranteed with learning sequences ρ_t that tend to 0 with occasional disturbances, see [2].
- Though the results are for $t \to \infty$ only, their impact can be seen in practical examples, e.g. in the travelling salesman problem where a shortest tour through given locations is to be found, see Fig. 2.

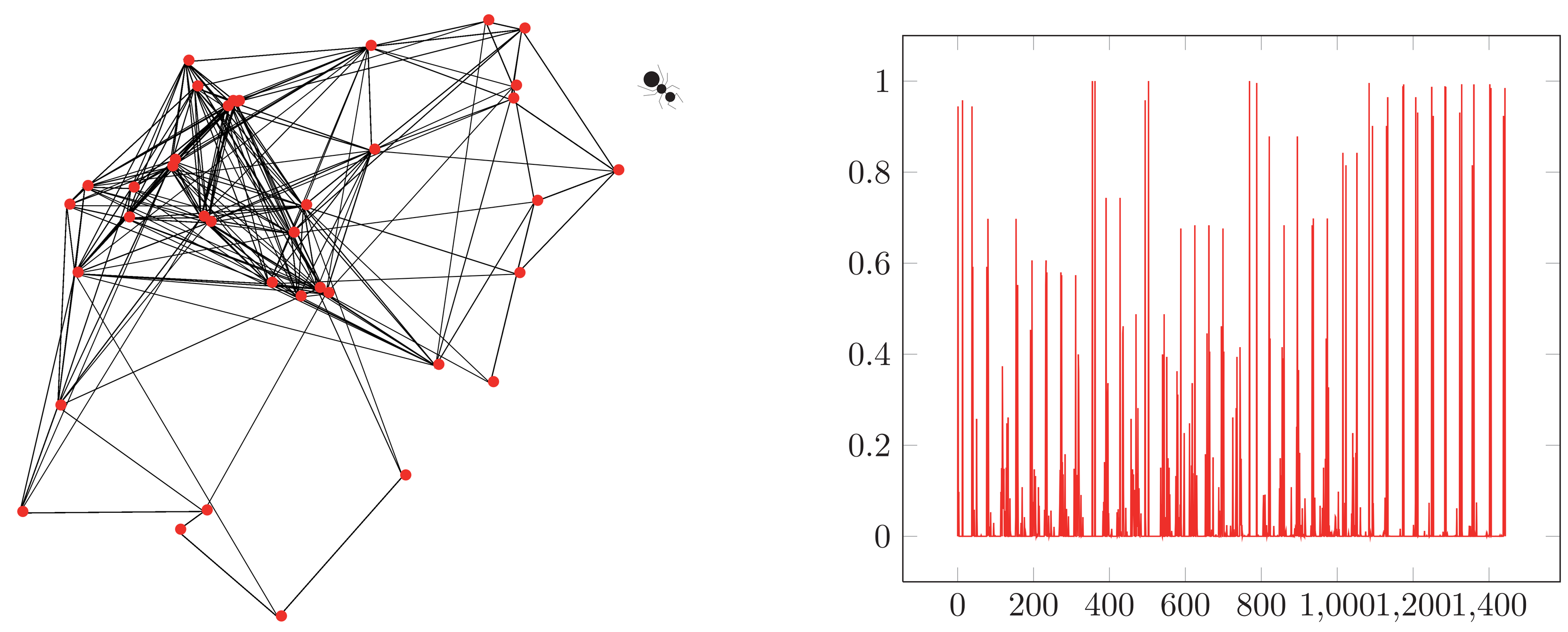

Figure 2: A travelling salesman wants to travel through 38 cities (red dots) along a shortest tour. These are randomly sampled from the 1046 possible connections between cities. Here we used a constant learning rate $\rho = 0.3$. After $t = 100$ iterations, the paths sampled still vary, the model Π_{100} consists of the red probabilities for each of the 1406 arcs indicated on the right hand side.

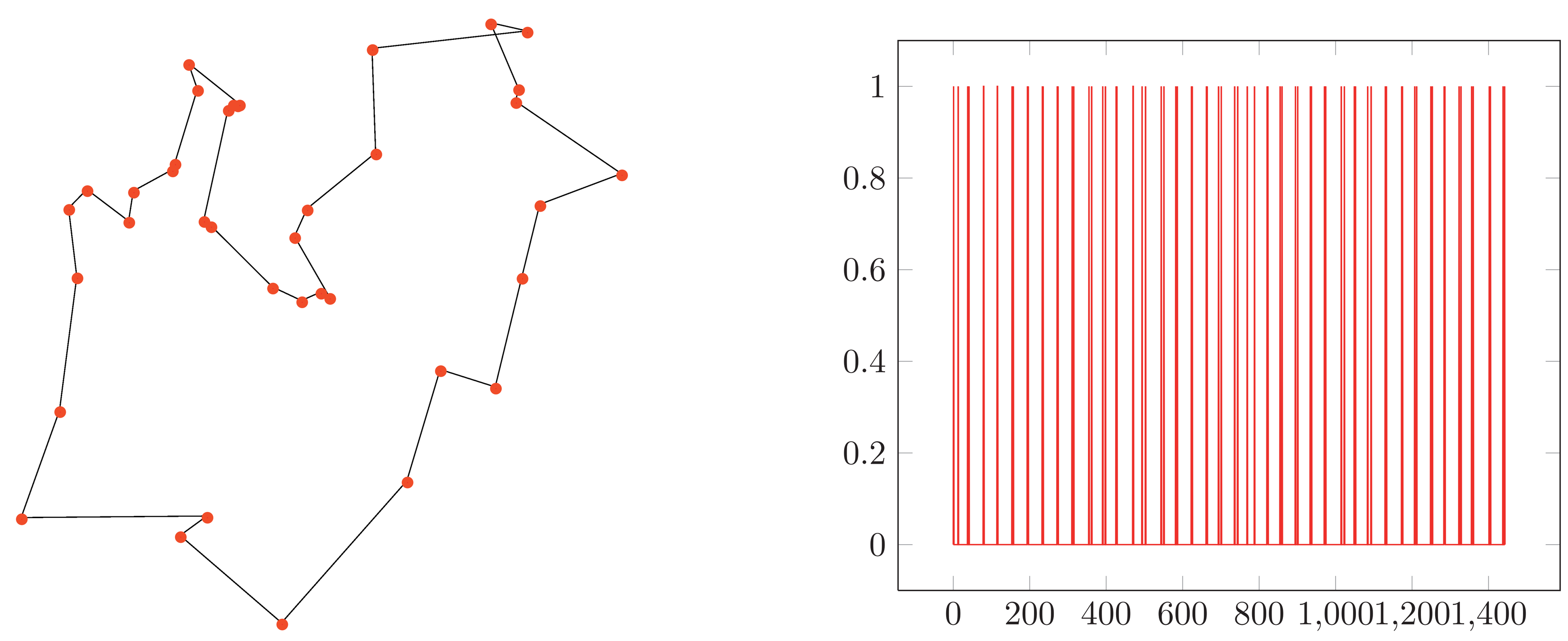

Figure 3: After 200 hundred iterations, all sampled tours coincide, the model has become a one point measure on this (slightly suboptimal) solution

[1] Wu, Zijun, "Model-based heuristics for combinatorial optimization: a mathematical study of their asymptotic behavior", Dissertation, Fakultät für Mathematik / Informatik und Maschinenbau, Technische Universität Clausthal April 2015
[2] Z. Wu, and M. Kolonko, "Asymptotic Properties of a Generalized Cross Entropy Optimization Algorithm" , IEEE Transactions on Evolutionary Computation, 18(2014), 1-16, DOI 10.1109/TEVC.2014.2336882