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## Overview of research results

In the following I will give a brief overview about my most important scientific achievements. Most of these results are related to the rigorous analysis of problems from fluid mechanics (incompressible as well as compressible models) but I also worked on more theoretical questions.

**Elliptic regularity theory.** In my PhD thesis I studied *variational problems with non-standard growth conditions*. I was able to significantly relax the assumptions on the growth rates of the integrand being necessary for the regularity of minimisers, see [1].
Still being interested in elliptic regularity theory my attention was drawn eventually to the *maximal regularity for the p-Laplacian system*. In a collaboration with A. Cianchi (Florence), L. Diening (Bielefeld), T. Kuusi (Helsinki) and S. Schwarzacher (Prague) we derived a flexible, comprehensive approach to gradient bounds for the p-Laplacian system based on a classical operator of harmonic analysis, cf. [4]. Our theory recovers a broad class of norm-estimates and augments the available literature. Based on this we turned to the investigation of boundary regularity. We derived a global maximal regularity estimate for the p-Laplacian system under minimal assumptions on the regularity of the boundary of the domain [5]. The sharpness of our result is demonstrated by a family of apropos examples.

**Function spaces.** Motivated mainly by problems from regularity theory and fluid mechanics I have been working on questions on embeddings for function spaces. Together with L. Diening (Bielefeld) and F. Gmeineder (Konstanz) we investigated a trace embedding for *functions of bounded* $\mathbb{A}$*-variation* (where $\mathbb{A}$ is a $\mathbb{C}$-elliptic operator), which gives a new point of view for the classical spaces $\mathrm{BV}$ and $\mathrm{BD}$, see [6].
More recently, I developed together with A. Cianchi (Florence) a complete theory for *function spaces built upon symmetric gradients* (the symmetric gradient of a vectorial function $\mathbf{u}$ is given by $\varepsilon(\mathbf{u}) = \frac{1}{2}\big(\nabla\mathbf{u} + \nabla\mathbf{u}^\top\big)$) and we proved optimal Sobolev embeddings in this context [3]. We were able to prove that these embeddings are equivalent to classical Sobolev embeddings (involving the full gradient $\nabla\mathbf{u}$) in the very general framework of rearrangement invariant function spaces despite the failure of Korn's inequality (which allows to control $\nabla\mathbf{u}$ by $\varepsilon(\mathbf{u})$).

**Generalised Newtonian fluids.** One of my main achievements in my time as a postdoc was the development of a *solenoidal Lipschitz truncation* (approximation of a divergence-free Sobolev function by a divergence-free Lipschitz continuous one such that both are equal up to a small set of small Lebesgue measure) – in the elliptic and parabolic setting (together with L. Diening, Bielefeld). This is a fundamental tool in the existence theory for incompressible generalized Newtonian fluids. In fact, new classes of fluids (like Prandtl-Eyring fluids) could be considered and the new approach highly simplified the existing methods. These results let to my first research monograph [2].

**Stochastic Navier–Stokes equations.** Stochastic partial differential equations (SPDEs), in particular those arising from fluid mechanics, are a main focus of my research. Together with M. Hofmanová (Bielefeld) and E. Feireisl (Prague) I started a systematic investigation of *stochastic compressible Navier–Stokes equations*. This advances the theory of SPDEs in fluid mechanics considerably and finally led to a research monograph [10]. A highlight is our result on the long-time behaviour of stochasically forced compressible fluid flows which shows a certain regularisation effect in comparison to the deterministic case, see [12]. A second main achievement is our analysis of the low-Mach number limit of the compressible system, see [9].
Using methods from stochastic analysis we investigated eventually the deterministc Euler equations and proposed a new approach to its well-posedeness, cf. [11]: We were able to construct a solution operator which enjoys the semi-group property.

**Fluid-structure interaction.** Together with S. Schwarzacher (Prague) we studied the Navier–Stokes equations governing the motion of an isentropic compressible fluid in three dimensions interacting with a *flexible shell*. The latter one constitutes a moving part of the boundary of the physical domain. The deformation is modeled by a linearised version of *Koiter's elastic energy* which leads to a fourth order hyperbolic equation. We were able to show the long-time existence of weak solutions to the corresponding coupled system of PDEs, see [13]. Very recently, we were able to generalise this result to heat-conducting fluids and fully nonlinear elastic structures (that is, the original model by Koiter), see [14]. This model is energetically closed and, consequently, we were able to show that even weak solutions satisfy an energy equality.

**Numerical analysis.** I have been interested in the numerical approximation of nonlinear PDEs in the elliptic, parabolic and stochastic setting. In particular, I studied finite-element based space-time approximations for *parabolic problems with low regularity* [7] and *stochastic Navier–Stokes equations* [8]. In both cases I proved optimal convergence rates with respect to the discretisation parameters. In the case of stochastic Navier–Stokes equations this improves previous results by A. Prohl regarding the convergence rate in time. This is a critical aspect for stochastic PDEs on account of the low time regularity of the driving Wiener process – for Navier–Stokes equations this is further complicated due to the quadratic nonlinearity and the divergence-free constraint.

# References


[1] D. Breit, **Calc. Var. & PDE** 44, 101–129. ('12)

[2] D. Breit, Elsevier/Academic Press, 286 pp. ('17)

[3] D. Breit, A. Cianchi, **Adv. Math.** 391, 107954. (2021)

[4] D. Breit, A. Cianchi, L. Diening, T. Kuusi, S. Schwarzacher, **J. Math. Pures Appl.** 114, 146–190. ('18)

[5] D. Breit, A. Cianchi, L. Diening, S. Schwarzacher, **Arch. Rational Mech. Anal.** DOI:10.1007/s00205-021-01712-w

[6] D. Breit, L. Diening, F. Gmeineder, **Anal. PDE** 13, 559–594. ('20)

[7] D. Breit, P. R. Mensah, **IMA J. Num. Anal.** 40, 2505–2552. ('20)

[8] D. Breit, A. Dodgson, **Numer. Math.** 147, 553–578. ('21)

[9] D. Breit, E. Feireisl, M. Hofmanová, **Arch. Rat. Mech. Anal.** 222, 895–926. ('16)

[10] D. Breit, E. Feireisl, M. Hofmanová, De Gruyter, 344 pp. ('18)

[11] D. Breit, E. Feireisl, M. Hofmanová, **Arch. Rat. Mech. Anal.** 235, 167–194. ('20)

[12] D. Breit, E. Feireisl, M. Hofmanová, B. Maslowski, **Prob. Theory Relat. Fields** 174, 981–1032. ('19)

[13] D. Breit, S. Schwarzacher: **Arch. Rat. Mech. Anal.** 228, 495–562. ('18)

[14] D. Breit, S. Schwarzacher, to appear in **Ann. Sc. Norm. Super. Pisa** Preprint at arXiv:2101.00824v1