Repeatedly trivial local maximum degree induced subgraphs of iterated line graphs

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Abstract

If a vertex of a given graph G has no neighbours of higher degree, we call it a local maximum degree vertex. Such a vertex always exists, e.g. any vertex of maximum degree. For the sequence of iterated line graphs $(L^k(G))_{k=0,1,\dots}$ with $L^0(G) = G$ and $L^{j+1}(G) = L(L^j(G))$ we study the subgraphs $\mathcal{LM}(L^k(G))$ induced by the respective sets of local maximum degree vertices. We focus on the situation where those subgraphs are repeatedly trivial, i.e. they consist only of a single vertex. Let $m_0(G)$ be the smallest number k such that $\mathcal{LM}(L^j(G))$ is nontrivial. It is shown that for each $k \in \mathbb{N} \cup \{0\}$ a graph G exists with $m_0(G) = k$. Further, an algorithm for the efficient determination of m_0 is presented.

Keywords: iterated line graph, maximum degree.

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