TU Clausthal

Solving the Traveling Tournament Problem by Packing Three-Vertex Paths

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Motivation

The n-team Traveling Tournament Problem (TTP) is an NP-hard sports scheduling problem that was inspired by the real-life problem of optimizing Major League Baseball schedules to reduce team travel. The TTP has attracted a significant amount of research, with a set of benchmark instances whose solutions are often found after weeks of computation on high-performance machines.

Main Result

We approach the TTP using graph theory, and determine a simple "canonical" schedule in which each team's three-game road trips match up with the underlying graph's minimum-weight P_3-packing. Starting with this schedule and applying two simple heuristics, we obtain tournament schedules for five benchmark TTP instances that beat all previously-known solutions.

Theoretical Contributions

In the TTP, the output is a double round-robin schedule that minimizes the total sum of distances traveled by all teams as they move from city to city, subject to several natural constraints to ensure balance and fairness. The TTP is similar to the Traveling Salesman Problem, only much harder! Besides Integer Programming and Constraint Programming, there are various approaches to find near-optimal solutions for the TTP.

We propose a three-phase approach to solving hard TTP instances: in Phase 1, a constructive procedure based on P_3-packings is used to produce an initial feasible schedule. In Phase 2, a simple local pairwise-swapping procedure attempts to improve this solution. In Phase 3, we take the solution from the previous phase and apply a "hybrid" algorithm (Goerigk and Westphal 2012) that uses heuristics such as tabu search to output a final feasible n-team tournament schedule.

This three-phase approach finds new solutions to five TTP benchmark sets: Galaxy22, Galaxy28, Galaxy34, Galaxy40, and NFL28

Results of the Three-Phase-Approach

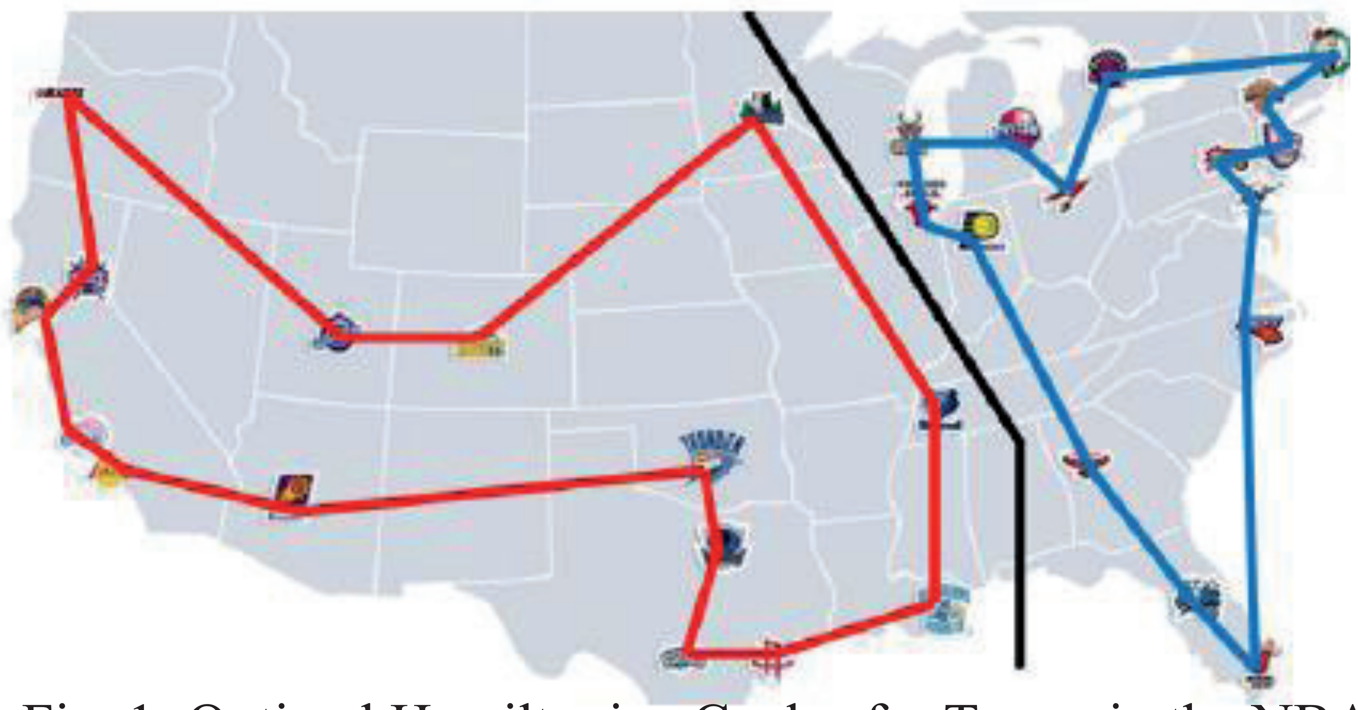

Fig. 1: Optimal Hamiltonian Cycles for Teams in the NBA

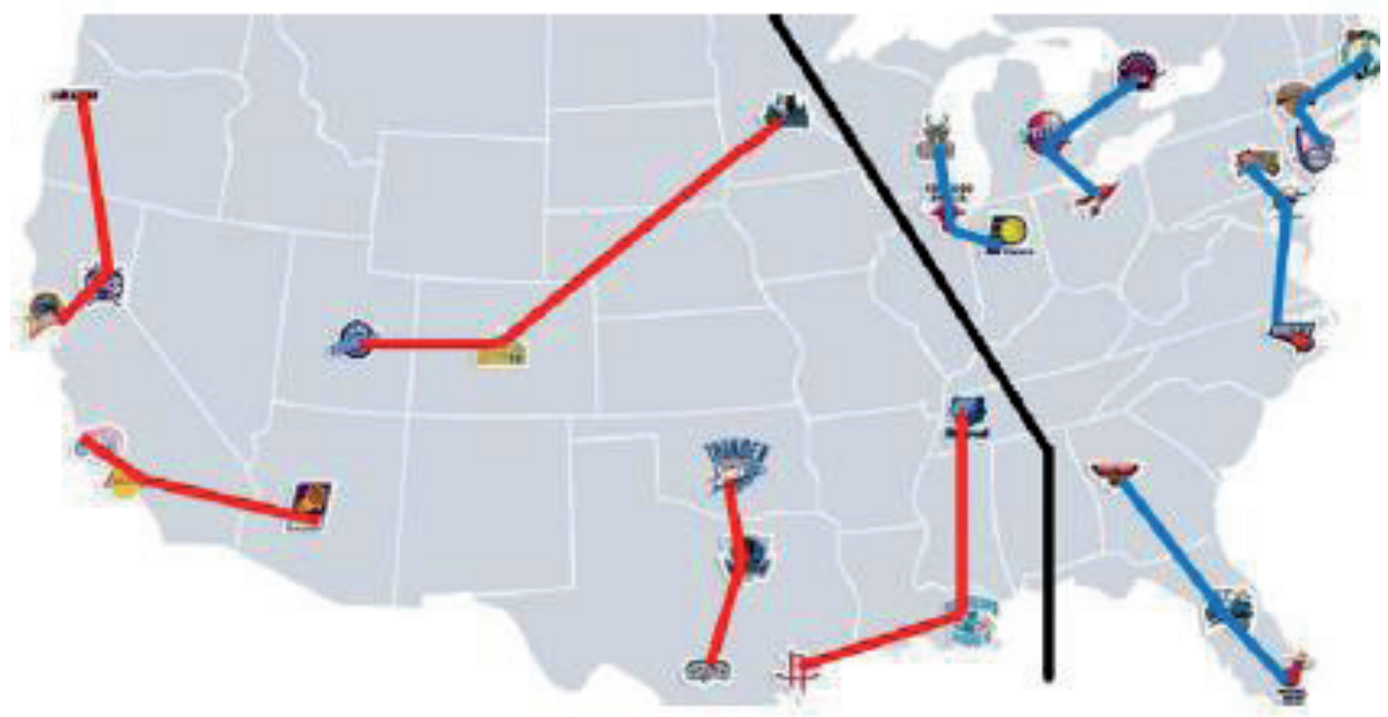

Fig. 2: Optimal P_3-packings for Teams in the NBA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_1	$\mathbf{t_{10}}$	$\mathbf{t_3}$	t_2	t_{10}	t_3	$\mathbf{t_2}$	$\mathbf{t_5}$	$\mathbf{t_6}$	t_4	t_5	t_6	$\mathbf{t_4}$	$\mathbf{t_8}$	$\mathbf{t_9}$	t_7	t_8	t_9	$\mathbf{t_7}$
t_2	$\mathbf{t_3}$	$\mathbf{t_{10}}$	$\mathbf{t_1}$	t_3	t_{10}	t_1	$\mathbf{t_6}$	t_4	t_5	t_6	$\mathbf{t_4}$	$\mathbf{t_5}$	$\mathbf{t_9}$	t_7	t_8	t_9	$\mathbf{t_7}$	$\mathbf{t_8}$
t_3	t_2	t_1	t_{10}	$\mathbf{t_2}$	$\mathbf{t_1}$	$\mathbf{t_{10}}$	t_4	t_5	t_6	$\mathbf{t_4}$	$\mathbf{t_5}$	$\mathbf{t_6}$	t_7	t_8	t_9	$\mathbf{t_7}$	$\mathbf{t_8}$	$\mathbf{t_9}$
t_4	$\mathbf{t_9}$	$\mathbf{t_8}$	$\mathbf{t_7}$	t_9	t_8	t_7	$\mathbf{t_3}$	$\mathbf{t_2}$	$\mathbf{t_1}$	t_3	t_2	t_1	$\mathbf{t_{10}}$	$\mathbf{t_6}$	t_5	t_{10}	t_6	$\mathbf{t_5}$
t_5	t_7	$\mathbf{t_9}$	$\mathbf{t_8}$	$\mathbf{t_7}$	t_9	t_8	t_1	$\mathbf{t_3}$	$\mathbf{t_2}$	$\mathbf{t_1}$	t_3	t_2	t_6	$\mathbf{t_{10}}$	$\mathbf{t_4}$	$\mathbf{t_6}$	t_{10}	t_4
t_6	t_8	t_7	$\mathbf{t_9}$	$\mathbf{t_8}$	$\mathbf{t_7}$	t_9	t_2	t_1	$\mathbf{t_3}$	$\mathbf{t_2}$	$\mathbf{t_1}$	t_3	$\mathbf{t_5}$	t_4	t_{10}	t_5	$\mathbf{t_4}$	$\mathbf{t_{10}}$
t_7	$\mathbf{t_5}$	$\mathbf{t_6}$	t_4	t_5	t_6	$\mathbf{t_4}$	t_{10}	$\mathbf{t_9}$	$\mathbf{t_8}$	$\mathbf{t_{10}}$	t_9	t_8	$\mathbf{t_3}$	$\mathbf{t_2}$	$\mathbf{t_1}$	t_3	t_2	t_1
t_8	$\mathbf{t_6}$	t_4	t_5	t_6	$\mathbf{t_4}$	$\mathbf{t_5}$	$\mathbf{t_9}$	t_{10}	t_7	t_9	$\mathbf{t_{10}}$	$\mathbf{t_7}$	t_1	$\mathbf{t_3}$	$\mathbf{t_2}$	$\mathbf{t_1}$	t_3	t_2
t_9	t_4	t_5	t_6	$\mathbf{t_4}$	$\mathbf{t_5}$	$\mathbf{t_6}$	t_8	t_7	$\mathbf{t_{10}}$	$\mathbf{t_8}$	$\mathbf{t_7}$	t_{10}	t_2	t_1	$\mathbf{t_3}$	$\mathbf{t_2}$	$\mathbf{t_1}$	t_3
t_{10}	t_1	t_2	$\mathbf{t_3}$	$\mathbf{t_1}$	$\mathbf{t_2}$	t_3	$\mathbf{t_7}$	$\mathbf{t_8}$	t_9	t_7	t_8	$\mathbf{t_9}$	t_4	t_5	$\mathbf{t_6}$	$\mathbf{t_4}$	$\mathbf{t_5}$	t_6

Fig. 3: Canonical Tournament Schedule for n=10 teams

n-team TTP Instance	Previously Best-Known Bound	Results after Phase 2	Results after Phase 3	Percentage Improvement
Galaxy22	35,467	35,014	**33,901**	4.42%
Galaxy28	77,090	76,518	**75,276**	2.35%
Galaxy34	146,792	145,165	**143,298**	2.38%
Galaxy40	247,017	245,052	**241,908**	2.07%
NFL28	609,788	613,574	**589,123**	3.39%